A Model of Dynamic Associative Memory

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ABSTRACT

A model of dynamic associative memories is proposed in this paper. The aim is to find all stored patterns, and to distinguish the stored and the spurious patterns. Aihara used chaotic neurons and showed that his model has a nonperiodic associative dynamics. In his model, however, it is difficult to distinguish the stored patterns from the others, because the state of the network changes continually. We propose such a new model of neurons that each neuron changes its output to the other when the accumulation of its internal state exceeds a certain threshold. By computer experiments, we show that the state of the network stays at the stored pattern for a while and then travels around to another pattern, and so on. Furthermore, when the number of the stored patterns is small, the stored and the spurious patterns can be distinguished using interval of the network staying these patterns.

1. Introduction

An associative memory is a hopeful application of neural networks(NNs). Connection weights are adjusted so that patterns are memorized on equilibrium states. In this paper, the recurrent neural network(RNN), in which all neurons are interconnected through a superposed auto-correlation synaptic matrix, is basically taken into account. The chaotic neural network that uses chaotic neurons constituting the RNN shows the nonperiodic associative dynamics [1],[2]. However, in this network, retrieval frequency of the stored patterns is small and it is difficult to distinguish the unstored patterns from the stored patterns. The purpose of this paper is to organize such a network that its state travels around the stored patterns and stays at each of them for a while.

2. Chaotic Neural Networks

2.1. Recurrent Neural Network

The associative memory by the RNN is described here. A neuron is connected with all the other neurons. Let the internal state and the output of the ith neuron at the tth transition cycle be $u_i(t)$ and $x_i(t)$, respectively. Network transition is formulated as follows:

$$u_i(t) = \sum_{j=1}^{N} w_{ij} x_j(t-1) \tag{1}$$

$$x_i(t) = f(u_i(t)) \tag{2}$$

where

$$f(u) = \begin{cases} 1 & u \geq 0 \\ -1 & u < 0 \end{cases} \tag{3}$$

Here, w_{ij} expresses the connection weight from the jth neuron to the ith neuron. The weights are de-

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termined according to the following symmetric autocorrelation matrix of the stored patterns:

$$w_{ij} = \frac{1}{M} \sum_{m=1}^{M} x_i^m x_j^m. \tag{4}$$

Due to interactions among neurons through these interconnections, the network shows self-associative dynamics.

The vector $x(t) = (x_1(t), x_2(t), ..., x_N(t))^t$ represents the network state, at the same time, it is called 'pattern' in this paper.

2.2. Chaotic Neural Networks

The dynamic associative memory proposed in [1], [2] is briefly described here for comparison. Chaotic neuron models proposed by Aihara can be used as constituent elements of the RNN, which is called "chaotic neural networks". The dynamics of the ith chaotic neuron is described as follows:

$$x_i(t+1) = f\left(\eta_i(t+1) + \zeta_i(t+1)\right) \tag{5}$$

where $x_i(t+1)$ is the neuron output with an analog value between 0 and 1, $f(u)$ is the sigmoid function, and $\eta_i(t+1)$ and $\zeta_i(t+1)$ are internal state terms for the feedback inputs from the neurons in the network and refractoriness, respectively, as described in the following.

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{N} w_{ij} x_j(t-d) \tag{6}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \tag{7}$$

where k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. a_i denotes the sum of the threshold and the temporally constant external inputs to the ith neuron.

This network behaves as a dynamic associative memory, in which the network state travels around the stored patterns. However, it is not always guaranteed that the network state stays the stored patterns for a while. For this reason, it is hard to extract the stored patterns. In Sec.4, simulation using this model will be omitted.

3. Proposed Dynamic Associative Memory

3.1. A New Neuron Model

We use a new type of neurons instead of chaotic neurons as constituent elements of the RNN.

In the RNN, after the network state is attracted to an equilibrium state, the state does not change. In order to get out from the equilibrium state, we propose a new model, whose output is reversed when the network stays at the same state for a while. This is done by the following.

After $u_i(t)$ and $x_i(t)$ are calculated by Eqs.(1) and (2), the internal state is accumulated as follows:

$$y_i(t) = y_i(t-1) + u_i(t) \tag{8}$$

where $y_i(t)$ expresses the accumulation of the internal state. When $|y_i(t)|$ exceeds some threshold, the output turns to the reverse, and $y_i(t)$ is reset to zero. This is expressed by.

If $|y_i(t)| \geq h$,

then

$$x_i(t) = -x_i(t) \tag{9}$$

$$y_i(t) = 0 \text{ (reset)} \tag{10}$$

Set $t = t + 1$, and return to Eq.(1).

The connection weights are determined by Eq.(4).

3.2. Characteristics of Proposed Network

By using this type of neurons as constituent elements of the RNN, we may expect the dynamics as described in the following.

1) When the network state is attracted to one of stored patterns, absolute value of $y_i(t)$ continues increasing because the value of internal state becomes the same in this period.

2) After the network state stays at the equilibrium state for a while, it goes away from this equilibrium state. In an equilibrium state, that is a stable pattern, the state of neurons having large internal state

tend to be reversed earlier. Since these neurons are important in this pattern, by reversing these neurons states, the network state can escape from this patterns.

3) The threshold h plays an important role in this network. In Eq.(8), $y_i(t)$ is determined by the initial value $y_i(t_0)$ at the begining of the equilibrium state, and the internal state in this state. Furthermore, $y_i(t_0)$ is determined in the transition process from one equilibrium state to the other. Therefore, it may be rather randomized. On the other hand, the internal state is determined by a set of the stored patterns, and is closely related to the pattern stability.

When the threshold h takes a small value, effect of $y_i(t_0)$ will be relatively large. On the other hand, the internal state will be dominant in $y_i(t)$ for the high threshold. This means that the network state transition will be randomized by using the low threshold, and will be rather deterministic with the high threshold.

4) After it goes away, the output is difficult to be reversed, because $|y_i(t)|$ is not accumulated in the nonequilibrium state. Therefore, the network state attracted to the equilibrium as the same as the RNN.

4. Simulation Results

4.1. Fundamental Dynamics

A mutually connected NN, having $10 \times 10 = 100$ neurons, is used. Four patterns shown in Fig.1 are memorized. The weight matrix is calculated by Eq.(4).

The network state transition is simulated starting from the stored pattern (a). In Fig.3, the network state transition is investigated using the Hamming distances(HD), the number of neurons whose output differ, between the output pattern and the stored patterns (a), (b) and (c). From this result, the network state stays at one of the stored patterns(HD=0) during some period and in a minute it changes to the other stored patterns(HD=0) or their reversed patterns(HD=100).

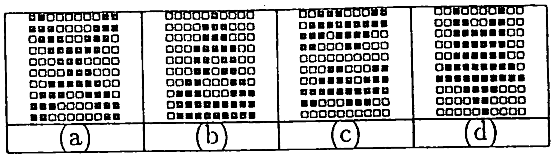


Fig. 1: Stored patterns (a) ~ (d).

4.2. Retrieval Characteristics

The network retrieves the stored patterns and their reverse patterns. Table 1 summarizes retrieval frequency of the stored patterns in the network with four different initial conditions shown in Figure 2. The network state is considered as the equilibrium state when it stays the same state more than one time. Frequency of equilibrium states are also shown in Table 1. This means that, for example, the stored pattern (a) is recalled 176 times, and it stays in the equilibrium state 141 times among them. From these results, it can be concluded that when the stored patterns are recalled, most of them stay the equilibrium states. The meaning of the threshold $h = 750$ will be discussed in Sec.4.3.

Table 2 shows transition frequency among the stored patterns and their reversed patterns. Table 3 shows correlations among the stored patterns. Comparing the transition frequency with these correlations, there is no relation between them.

Tables 1 and 2 show that retrieval characteristics depend on the initial conditions. In other words, the network can generate various retrieval characteristics depending upon the initial conditions.

4.3. Effect of Threshold on Dynamics

In the previous experiment, the threshold h for the accumulation of the internal state, whose average is

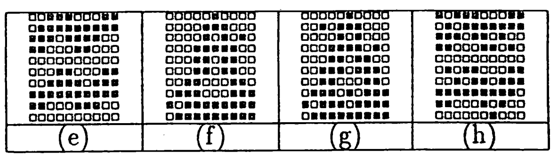


Fig. 2: Initial conditions (e) ~ (h).

Table: 1: Retrieval frequencies of stored and spurious patterns during 5000 iterations(h = 750)

Initial condition(e)

	Retrieval	equilibrium
Stored (a)	176	141
Stored (b)	1601	1314
Stored (c)	888	750
Stored (d)	176	145
Total	2841	2350
Suprious		1118

Initial condition(f)

	Retrieval	equilibrium
Stored (a)	113	94
Stored (b)	1342	1132
Stored (c)	952	782
Stored (d)	331	272
Total	2738	2280
Suprious		1249

Initial condition(g)

	Retrieval	equilibrium
Stored (a)	370	301
Stored (b)	1276	1046
Stored (c)	883	701
Stored (d)	223	188
Total	2752	2236
Suprious		1201

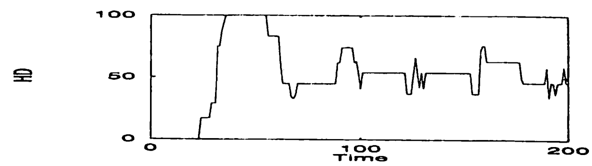
Initial condition(h)

	Retrieval	equilibrium
Stored (a)	85	69
Stored (b)	1032	854
Stored (c)	641	519
Stored (d)	1048	839
Total	2806	2281
Suprious		1042

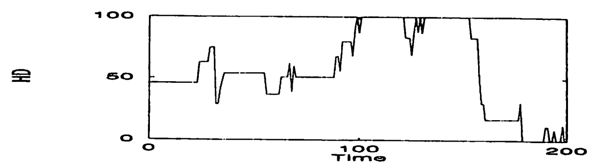
about 25 in the stored patterns, was fixed to 750. Next, effects of the threshold h on the dynamics is investigated. Figures 4(a) and 4(b) show effects of the threshold h on both the retrieval frequency and the transition frequency to other stored pattern, respectively. The former is roughly proportional to h, while the latter is inversely proportional to h. The reason is that, when the threshold is large, the interval in which a neuron don't reverse is long and the number of neurons, whose output are reversed at once is small.

4.4. Distinction between Stored and Spurious Patterns

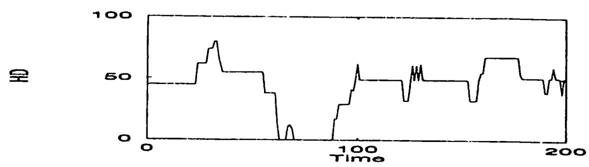
From Sec.4.2, the equilibrium states correspond to both the stored and the spurious patterns. Figure 5 shows the intervals in which the network state stays in the same patterns, that is the stored patterns and the spurious patterns. From this result, the stored and the spurious patterns can be distinguished by using h = 1500. In this simulation, the interval includes the following two states. First, the network state stays in the equilibrium state. Second, even though it escapes from the above state, it quickly returns to the same state.



Stored pattern (a)



Stored pattern (b)



Stored pattern (c)

Fig. 3: Hamming distances between the output pattern and the stored pattern (h = 750).

Table: 2: Transition frequencies among stored patterns and their reversed patterns during 5000 iterations(h = 750)

Initial condition(e)

	Stored				Reverse			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Sto.(a)	36	1	1	0	0	1	1	0
Sto.(b)	0	661	4	1	0	4	4	2
Sto.(c)	1	4	364	1	1	11	0	1
Sto.(d)	0	0	0	66	1	2	0	0
Rev.(a)	0	0	0	0	100	1	1	0
Rev.(b)	2	1	10	1	0	613	9	1
Rev.(c)	0	8	4	0	0	1	352	2
Rev.(d)	0	2	0	0	0	4	0	70

Initial condition(f)

	Stored				Reverse			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Sto.(a)	36	1	0	0	0	1	2	0
Sto.(b)	1	406	3	3	4	4	6	1
Sto.(c)	0	2	330	0	0	10	3	1
Sto.(d)	0	0	1	203	0	1	3	0
Rev.(a)	0	0	0	0	47	7	0	0
Rev.(b)	2	7	10	1	2	679	3	0
Rev.(c)	0	11	2	0	1	2	419	1
Rev.(d)	1	0	0	1	0	1	0	61

Initial conditon(g)

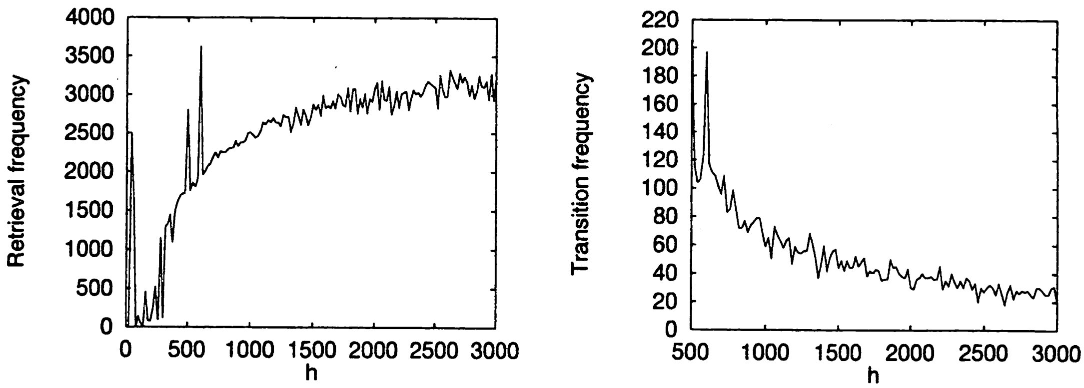
	Stored				Reverse			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Sto.(a)	161	1	0	1	1	1	3	2
Sto.(b)	5	565	0	2	2	3	6	0
Sto.(c)	2	2	234	1	0	5	1	1
Sto.(d)	0	4	0	97	1	1	1	0
Rev.(a)	0	0	3	0	125	2	0	1
Rev.(b)	2	3	6	2	0	447	1	1
Rev.(c)	0	7	0	0	2	1	444	2
Rev.(d)	0	2	2	1	0	2	0	77

Initial condition(h)

	Stored				Reverse			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Sto.(a)	26	0	0	0	0	1	1	0
Sto.(b)	1	386	1	1	1	3	1	1
Sto.(c)	1	2	328	2	0	6	1	0
Sto.(d)	0	1	3	119	0	1	3	0
Rev.(a)	0	1	0	0	39	1	0	0
Rev.(b)	0	1	6	3	0	443	3	3
Rev.(c)	0	3	0	1	1	3	170	1
Rev.(d)	0	1	2	0	0	1	0	708

Table: 3: Correlations among stored patterns(a) ~ (d)

	Sto.(a)	Sto.(b)	Sto.(c)	Sto.(d)
Stored (a)	1.00	0.08	0.10	0.06
Stored (b)	0.08	1.00	-0.02	0.06
Stored (c)	0.10	-0.02	1.00	0.08
Stored (d)	0.06	0.06	0.08	1.00



(a)Retrieval frequency (b)Transition frequency

Fig. 4: Effects of threshold h on both retrieval and transition frequencies.

5. Conclusions

A model of the dynamic associative memory has been proposed in this paper. A new model of neuron changes its state after staying in the same pattern for a while. The network state can travel around the stored patterns and can stay at them during some period. The network state transition among the stored patterns has no relation to the correlation among them. When the threshold for the accumulation of the internal state is large, the retrieval frequency is large while the transition frequency is small. The stored and spurious patterns can be distinguished in some cases based on the intervals, in which the network state stays in the same patterns.

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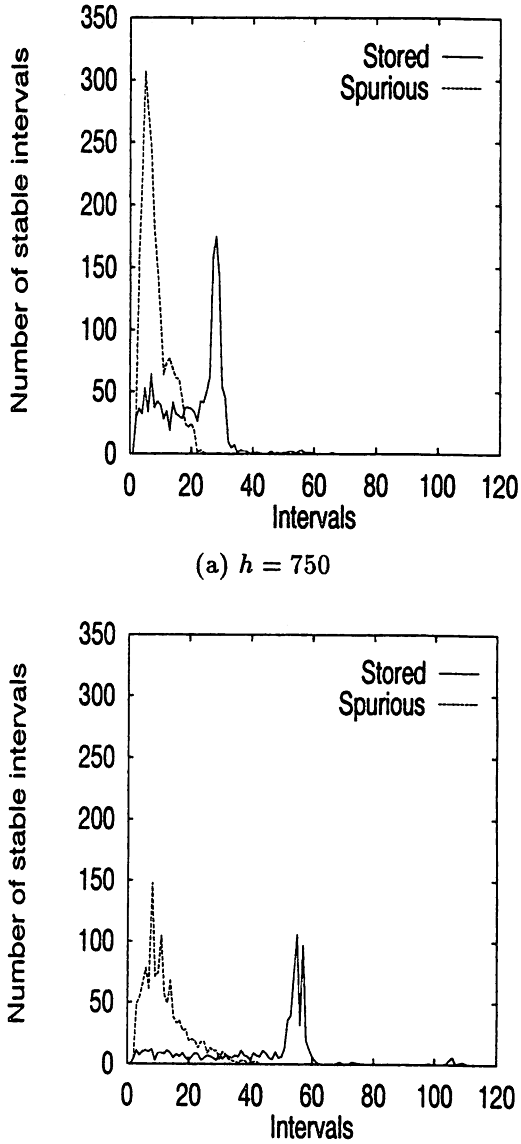

(a) $h = 750$

(b) $h = 1500$

Fig. 5: Intervals in which network state stays in the same patterns.