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Optimum Order Assignment on Numerator and Denominator for IIR Adaptive Filters Adjusted by Equation Error*

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SUMMARY For system identification problems, such as noise and echo cancellation, FIR adaptive filters are mainly used for their simple adaptation and numerical stability. When the unknown system is a high-Q resonant system, having a very long impulse response, IIR adaptive filters are more efficient for reduction in the order of a transfer function. One way to realize the IIR adaptive filter is a separate form, in which the numerator and the denominator are separately realized and adjusted. In the actual applications, the order of the unknown system is not known. In this case, it is very important to estimate the total order and the order assignment on the numerator and the denominator. In this paper, effects of the order estimation error on the residual error are investigated. In this form, indirect error evaluation called "equation error" is used. Through theoretical and numerical investigation, the following results are obtained. First, under estimation of the order of the denominator causes large degradation. Second, over estimation can improve the performance. However, this improvement is saturated to some extent due to cancellation of the redundant poles and zeros. Third, the system identification error is proportional to the equation error as the adaptive filter approaching the optimum. Finally, there is possibility of recovering from the unstable state as the order assignment approaches to the optimum in an adaptive process using the equation error. Computer solutions are provided to aid in gaining insight of the order assignment and stability problem.

key words: *IIR adaptive filter, equation error, order assignment*

1. Introduction

In case of Infinite Impulse Response (IIR) adaptive filters the number of taps can be reduced drastically. It is to be noted here that a number of IIR or IIR like methods have been proposed both in adaptive signal processing and system identification community [1]–[5]. However, the class IIR itself comes off a more general ARMAX model family [6]. Though the equation and the output error methods are common in IIR class, the output error corresponds to actual transfer function error, whereas the equation error is a filtered version of it. This will be made more clear in Sect. 2.

In applying the IIR adaptive filters, it is very important to optimize order assignment on the numerator and the denominator. Especially, when the total number of coefficients is limited, performance of the IIR adaptive filter is very sensitive to this order assignment.

There exist some methods to estimate optimum order for non adaptive system identification problems; one of them is Akaike's Information Criterion (AIC) [7], which uses the final prediction error to calculate optimum order in least square problems. In other cases, e.g., in speech processing applications [8],[9], which generally correspond to a particular prediction error category, expressed later in Eq. (9) in Sect. 2, other model order estimation methods also have been proposed [8],[9]. However, it seems that investigations around the "separated realization of the IIR" adaptive filters, which has been defined in Eq. (7) in Sect. 2 are rare. Methods for estimating optimum tap assignment for a given number of total taps may not be easy available for this particular structure of the IIR filter.

In this paper, effects of order assignment in the separate realization of the IIR adaptive filter is investigated. Over and under estimation for the filter order are also discussed. Furthermore, efficiency of the equation error to evaluate the performance of the filter is investigated. Finally, the stability problem in a process of finding the optimum order assignment is discussed. Recursive least square (RLS) algorithm is employed. The system identification problem is taken into account.

2. Differences among IIR Adaptive Filters

To define differences among IIR adaptive filters [6] we write input-output relation of an unknown system as

$$d(n) = \mathbf{A}^T\mathbf{d}(n-1) + \mathbf{B}^T\mathbf{u}(n) + e_m(n) \tag{1a}$$
$$= \mathbf{\Phi}^T\mathbf{X}(n) + e_m(n) \tag{1b}$$

where $\mathbf{A}$ and $\mathbf{B}$ are coefficient vectors and $\mathbf{d}(n-1)$ and $\mathbf{u}(n)$ are desired and input sample vectors, respectively. T is a transposed operation. $e_m(n)$ is measurement noise. $\Phi = [\mathbf{B}, \mathbf{A}]$, $\mathbf{X}(n) = [\mathbf{u}(n), \mathbf{d}(n-1)]$. The vectors $\mathbf{B}, \mathbf{A}, \mathbf{u}(n)$ and $\mathbf{d}(n-1)$ are defined as

$$\mathbf{B} = [b_0, ..., b_{M-1}]^T \tag{2a}$$
$$\mathbf{A} = [a_1, ..., a_{N-1}]^T \tag{2b}$$
$$\mathbf{u}(n) = [u(n), ..., u(n-M+1)]^T \tag{2c}$$
$$\mathbf{d}(n-1) = [d(n-1), ..., d(n-N+1)]^T \tag{2d}$$

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Since the white unmeasurable $e_m(n)$ in Eq. (1) is unpredictable, the above can be further modeled by

$$\hat{d}(n) = \hat{\mathbf{A}}^T\mathbf{d}(n-1) + \hat{\mathbf{B}}^T\mathbf{u}(n) \quad (3a)$$
$$= \hat{\mathbf{\Phi}}^T\mathbf{X}(n) \quad (3b)$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are estimated parameters. Now define prediction error,

$$e(n) = d(n) - \hat{d}(n) \quad (4)$$

The above equation can be rewritten using Eqs. (1b) and (3b) as

$$e(n) = [\mathbf{\Phi} - \hat{\mathbf{\Phi}}]^T\mathbf{X}(n) \quad (5)$$
$$= \tilde{\mathbf{\Phi}}^T\mathbf{X}(n) + e_m(n) \quad (6)$$

Tilde and hat have been used to denote the error of the estimated entities and estimation itself, respectively. Now, the contents of the information vector $\mathbf{X}(n)$, in prediction problem as expressed in the above equation, is important since they define the IIR adaptive filter classification. In IIR adaptive filters, $\mathbf{X}(n)$ may be defined as any of the following equations

$$\mathbf{X}(n) = [\mathbf{u}(n), \mathbf{d}(n-1)] \quad (7)$$
$$\mathbf{X}(n) = [\mathbf{u}(n), \hat{\mathbf{d}}(n-1)] \quad (8)$$

or even [8],[9]

$$\mathbf{X}(n) = [\hat{\mathbf{u}}(n), \mathbf{d}(n-1)] \quad (9)$$

It is important to note that Eqs. (7) – (9) express differences among the IIR adaptive filters. The prediction error $e(n)$ as defined in Eq. (6), is called "equation error" and the mean-square-equation-error is a quadratic function with a single global minimum [10]. Please note that this separate form of IIR filter, may also be expressed as a two-input, $d(n-1)$ and $u(n)$, single output $\hat{d}(n)$ model [6], where numerator and denominator coefficients are adjusted separately [11]. Now, the second case as defined in Eq. (8) corresponds to an "output error" method, which also may be stated as a direct form of IIR structure. It can be shown that the prediction error in this case [6]

$$e(n) = [1 - \mathbf{A}]^{-1}[\tilde{\mathbf{\Phi}}^T\mathbf{X}(n) + e_m(n)] \quad (10)$$

It can be observed from Eq. (10) that the prediction error sequence e(n), in this case, possesses poles and is an AR filtered version of the "equation error". It is to be noted here that the poles of this AR transfer function, $[1-\mathbf{A}]^{-1}$ are those of the actual process, which we want to estimate. Since mean square of the output error, is not a quadratic function of the parameters to be adjusted, it may have multiple local minima [10]. The third case is a special form that corresponds to Eq. (9). The information vector $\mathbf{X}(n)$ in this case contains a term $\hat{\mathbf{u}}(n)$, as shown in Eq. (9), which itself is an estimated parameter. Examples of this kind are found in speech processing applications e.g.,[8],[9].

The evaluation of equation error is different from the output error. Namely, it is indirect error evaluation. It has been shown that these two forms of error are equivalent in the sense that any of them can be used as the minimization criterion [12]. However, in the Sect. 3.2, it will be shown that the equation error does not represent exactly transfer function error. So, the equation error must be evaluated in comparison with the transfer function or the impulse response error.

3. Separate Realization of IIR Adaptive Filter

3.1 Network Structure

A block diagram of the separate realization of the IIR adaptive filter is shown in Fig. 1. $H(z)$ indicates a transfer function of the unknown system to be identified. $AF_a(z)$ and $AF_b(z)$ construct the denominator and numerator, respectively. $x(n)$ is noise to be canceled, for instance. The output of $H(z)$, denoted $d(n)$, is used as a desired response. It may be pointed out here that $[1-AFa(z)]^{-1}$ is copied to an all pole filter which is in cascade with $AFb(z)$, to get the output of the adaptive filter; however, this is not shown in the block diagram.

3.2 Equation Error Evaluation

The error given by Eq. (4) is the equation error. The z-transform of this error is derived in the following. Letting $D(z)$, $X(z)$, $Y_a(z)$, $Y_b(z)$, $Y(z)$, and $E(z)$ be z-transform of $d(n)$, $x(n)$, $y_a(n)$, $y_b(n)$, $y(n)$, and $e(n)$, respectively, we obtain

$$D(z) = H(z)X(z) \quad (11a)$$
$$Y_a(z) = AF_a(z)D(z) \quad (11b)$$
$$Y_b(z) = AF_b(z)X(z) \quad (11c)$$
$$Y(z) = Y_a(z) + Y_b(z) \quad (11d)$$
$$E(z) = D(z) - Y(z) \quad (11e)$$

By eliminating $D(z)$, $Y_a(z)$ and $Y_b(z)$, $E(z)$ can be expressed as

$$E(z) = [H(z) - H(z)AF_a(z) - AF_b(z)]X(z) \quad (12)$$

The ideal solution can be obtained by setting the inside of the bracket to be zero.

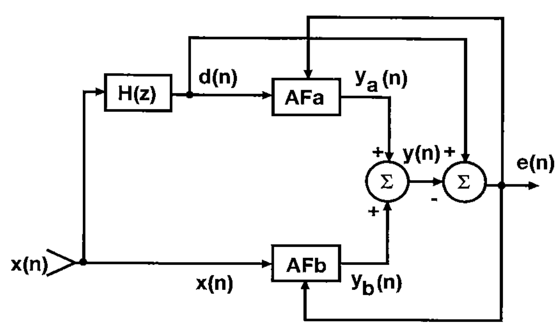


Fig. 1 Separate realization of IIR adaptive filter.

$$H(z) - H(z)AF_a(z) - AF_b(z) = 0 \tag{13}$$

From this condition, the following relation is obtained.

$$H(z) = \frac{AF_b(z)}{1 - AF_a(z)} \tag{14}$$

On the other hand, if the condition expressed in Eq. (13) cannot be satisfied, $H(z)$ should include an error term $\Delta H(z)$ in Eq. (12), and their relation is rewritten as

$$H(z) + \Delta H(z) = \frac{AF_b(z) + E(z)/X(z)}{1 - AF_a(z)} \tag{15}$$

$$\Delta H(z) = \frac{E(z)/X(z)}{1 - AFa(z)} \tag{16}$$

The above equation shows that the equation error $E(z)$ is weighted by $X(z)$ and $1 - AF_a(z)$ in the transfer function error criterion. Equation (16) indicates that not the equation error but a filtered version of it can exactly represent the transfer function error [15]. Possibility of finding the global minimum solution by using the equation error will be discussed in the later section.

3.3 Adaptive Filter Algorithm

Since the IIR adaptive filter discussed here has the separated numerator and denominator, the conventional algorithms for FIR adaptive filters can be used [5], [11], [14]. Recursive Least Square (RLS) algorithm [16] is used.

3.4 Stability Problem

Though the equation error does not possess any pole, the equation error adaptive filter itself may suffer from stability problem [6],[10],[13],[14]. However, the adaptation is not affected by unstable poles of the adaptive filter, which are given by $1/(1 - AF_a(z))$. So, adaptation can be continued even though some poles of the adaptive filter itself locate outside the unit circle, and finally the stable optimum solution may be found. This will be further discussed in Sect. 5.3.

4. Estimation and Assignment of Filter Order

4.1 Order Assignment

Filter order estimation is important for the system identification problem. Especially, when a sum of the numerator and denominator orders is limited in the adaptive filter, the optimum order assignment becomes very important from the relation given by Eqs. (14) and (16). Error property caused by under estimation of the order in the IIR adaptive filters is different from that of the FIR adaptive filters as shown in Eq. (16). The estimation error affects the filter performance with some weighting functions. High-Q filters will cause large error. In other words, the performance of the IIR filters is more sensitive to the order estimation than the FIR filters.

4.2 Error Criteria

The equation error is evaluated in this paper using the following relation.

$$E_{eq} = \frac{1}{K} \sum_{i=n_0}^{n_0+K-1} | e(i) |^2 \tag{17}$$

It is assumed that at $n = n_0$ the adaptation already converges. K is the number of error sample taken into account.

As discussed in Sect. 3.2, E_{eq} does not directly correspond to the transfer function error. Therefore, in simulation, efficiency of the equation error is evaluated by comparing the following error criteria.

$$E_{imp} = 10log_{10} \frac{\| \mathbf{h} - \mathbf{h}_{AF} \|^2}{\| \mathbf{h} \|^2} \tag{18}$$

$$\mathbf{h} = [h(0), h(1), ..., h(L-1)]^T \tag{19}$$

$$\mathbf{h}_{AF} = [h_{AF}(0), h_{AF}(1), ..., h_{AF}(L-1)]^T \tag{20}$$

$\| . \|$ indicates Euclidean norm. $h_H(n)$ and $h_{AF}(n)$ are impulse responses of $H(z)$ and $H_{AF}(z)$ shown below, respectively.

$$H_{AF}(z) = \frac{AF_b(z)}{1 - AF_a(z)} \tag{21}$$

5. Simulation and Discussions

5.1 Unknown System and Input Signal

The following transfer function is used for the unknown system.

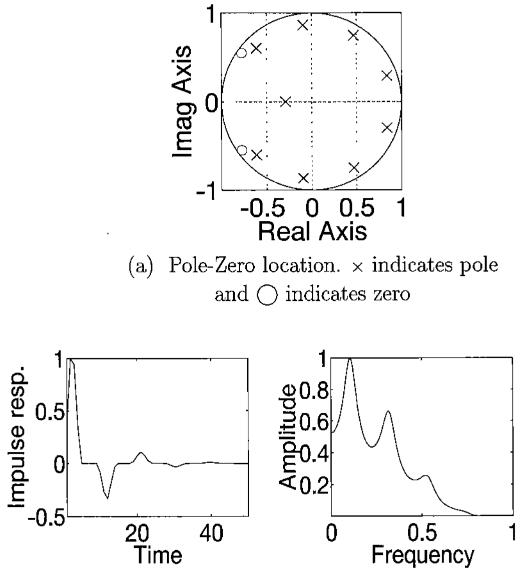
$$H(z) = \frac{1 + 1.5z^{-1} + 0.9025z^{-2}}{D(z)} \tag{22}$$

$$\begin{aligned} D(z) = [&1 - 0.9z^{-1} + 0.8z^{-2} - 0.7z^{-3} + 0.6z^{-4} \\ &- 0.5z^{-5} + 0.4z^{-6} - 0.3z^{-7} + 0.2z^{-8} \\ &+ 0.1z^{-9}] \end{aligned}$$

The impulse and the amplitude responses are shown in Figs. 2 (b) and (c), respectively. The sampling frequency is set to 2 Hz. The input signal is a white noise. The RLS algorithm with $\lambda = 0.95$ is used.

5.2 Effects of Order Assignment

From Eq. (22), it can be noted that the total order of the unknown system is 11th order, and the total number of coefficients is 13. By limiting these number to be invariant, effects of order assignment on the error criteria given in Sect. 4.2 are investigated.



(a) Pole-Zero location. × indicates pole and ○ indicates zero

(b) Impulse response (c) Frequency response

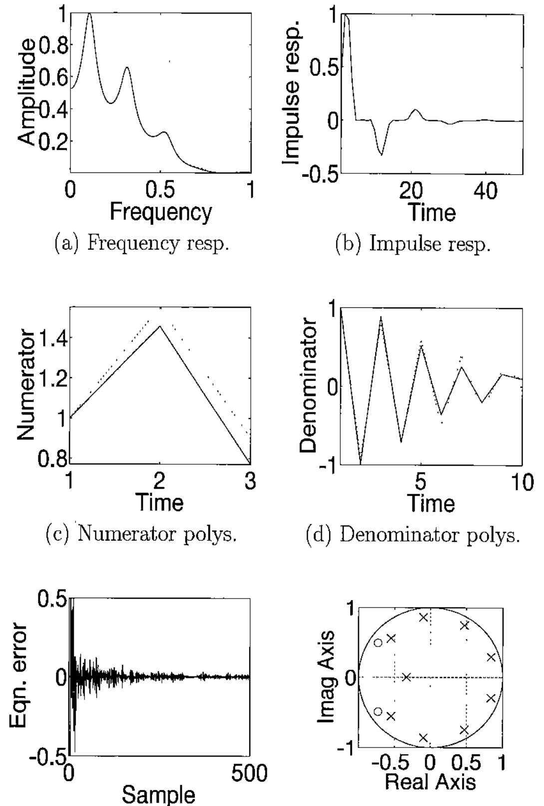
Fig. 2 Characteristics of unknown system.

Table 1 Error criteria for different tap ratios.

N/D	E_{eq} $\times 10^{-4}$	E_{imp} (dB)
12/1	31.82	−15.23
11/2	52.03	−14.47
10/3	52.0	−10.46
9/4	41.0	−8.97
8/5	33.0	−9.05
7/6	33.0	−9.13
6/7	32.0	−11.57
5/8	27.0	UNSTABLE
4/9	1.50	−33.02
3/10	**0.87**	**−43.86**
2/11	30.0	UNSTABLE
1/12	2.95	−21.73

Table 1 shows the simulation results. N/D means the ratio of the numbers of numerator and denominator coefficients. Adaptation was carried out independently for each ratio starting from zero initial coefficient values. Figure 3 shows results of the optimum ratio, obtained from Table 1, i.e., 3/10. Adaptive filter coefficients after 500 iterations, frequency and impulse responses calculated from those coefficients and the equation error are shown in Fig. 3 (c)(d), (a)(b) and (e) respectively. Also Fig. 3 (f) shows the pole-zero locations of the adaptive filter after 500 iterations. Solid and dotted lines correspond to the adaptive filter characteristics and the corresponding characteristics of the unknown system, respectively. The result shown in this figure indicates that a very satisfactory system identification is possible using an optimum N/D ratio.

From Table 1, it can be noted that the equation error E_{eq} can monotonously decrease toward the optimum assignment. Around the optimum assignment, E_{eq} is approximately proportional to E_{imp}. From these results, it can be concluded that the equation error can be used in searching for the optimum order assignment.



(a) Frequency resp. (b) Impulse resp.

(c) Numerator polys. (d) Denominator polys.

(e) Equation error (f) Pole-zero location. ×indicates pole ○ indicates zero

Fig. 3 Simulation results for 3/10 assignment. In (a) through (d), solid and dotted lines represent adaptive and unknown system respectively.

Table 2 Error criteria for different tap ratios with −20 dB measurement noise and 100 data samples.

N/D	E_{eq}	E_{imp} (dB)
10/3	0.0260	−11.14
9/4	0.0351	−9.87
8/5	0.0334	−9.25
7/6	0.0395	−9.43
6/7	0.0348	−11.10
5/8	0.0185	−14.28
4/9	0.0037	−27.23
3/10	**0.0018**	**−33.18**
2/11	0.0153	UNSTABLE

Table 2 shows a more severe case. In this case, the uncorrelated white measurement noise with a variance of −20 dB is added at the unknown system output. Furthermore, number of data was limited to only 100 samples. In this case, the optimum tap assignment can be still found.

5.3 Stability Analysis

In Table 1, the impulse response of the adaptive filter diverges in the cases of N/D=5/8 and 2/11. This

Table 3 Stability analysis in adaptation process. Ratio N/D is changed from 5/8 to 2/11.

N/D	5/8	4/9	3/10	2/11
E_{eq}	27.0×10^{-4}	3.28×10^{-4}	0.59×10^{-4}	4.84×10^{-4}
E_{imp}	Unstable →	Stable -23.11 dB	Stable -42.05 dB	Stable -27.88 dB
Iteration	0 ~ 500	501 ~ 1000	1001 ~ 1500	1501 ~ 2000

Table 4 Error criteria for different number of taps.

N/D	E_{eq} $\times10^{-7}$	E_{imp} (dB)
2/8	32000.0	−23.70
2/10	33000.0	−29.26
3/9	5269.8	−29.98
3/10	875.0	−43.86
5/12	141.0	−53.46
7/14	6.76	−70.82
9/16	0.04	−92.54
11/18	1.69	−75.54

problem is further investigated in the following condition. The ratio N/D initially starts from 5/8, and is successively changed every 500 iterations toward 2/11. 500 iterations has been determined in order to guarantee convergence by experience. The adjusted coefficients for N/D $= n/m$ are used as the initial guess for N/D $= (n-1)/(m+1)$. Simulation results are shown in Table 3.

The resulting error criteria are a little different from those in Table 1, due to the different initial guess. The filter falls into the unstable state in the ratio of 5/8. However, it can recover from the unstable state in the following adaptation using the ratios toward the optimum. This property of the equation error can guarantee the possibility to find the optimum order assignment in stable state.

On the contrary, the direct form, which uses the output error criterion, can not continue adaptation after the IIR filter falls into unstable state. Because the error diverges and hence can not be used for adaptation.

5.4 Effects of Increasing Filter Order

Another point to be optimized is the total order of the adaptive filter. In other words, the total number of the filter coefficients, that is N+D. In this section, effects of increasing N+D while maintaining the ratio N/D approximately invariant are investigated. Simulation results are shown in Table 4.

Obviously, by increasing N+D, the error criteria can be decreased. However, this improvement is saturated up to around −75 dB in E_{imp}. The reason can be explained as follows: High order transfer functions include redundant poles and zeros. In the adaptation process, they could be canceled to each other.

5.5 Convergence Rate and Tracking Speed

Convergence and tracking speeds depend on how to find an optimum order assignment in real time. An automatic finding method has not been proposed in this paper. However, it can be said that it is possible to find the optimum order assignment based on the equation error. The optimum solution can be found by changing the the order assignment so as to minimize the equation error starting from some order ratio. Furthermore, this method can track a time varying the unknown system.

6. Conclusions

The performance of the separate realization of the IIR adaptive filters, which is adjusted by using the equation error, has been discussed based on the order assignment. Once the total order of the adaptive filter is fixed, the performance is very sensitive to the order assignment. Around the optimum order assignment, the equation error is approximately proportional to the transfer function error. The adaptive filter can recover from the unstable state as its order assignment approaches toward the optimum. Therefore, it is possible to find the optimum order assignment using the equation error.

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