PERFORMANCE OF NLMS NOISE CANCELLER BASED ON CORRELATION BETWEEN SIGNAL AND NOISE

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ABSTRACT

In our paper, we investigate relations between noise canceller performance and signal-noise cross-correlation, based on theoretical analysis and computer simulation. Normalized LMS (NLMS) algorithm is employed. Uncorrelated, partially correlated, and correlated signal and noise combinations are taken into account. First, mathematical formulas for the spectrum of the canceller error are derived. Based on these results, the noise canceller performance can be predicted to some extent. Computer simulation is carried out, using real speech signal, white noise, real noise sound, multi-frequency signals, and their combinations. From the simulation , it is confirmed that the noise canceller performance is inversely proportional to the signal and noise cross-correlation, as expected by the theoretical analysis.

1 INTRODUCTION

The engineering community has paid a great attention to adaptive filtering problem in these last years, because of its applications in various fields such as noise canceller, system identification and others. From the litelatures, it is well known that noise canceller performance depends on the correlation between the signal and the noise.

The objective of our paper is to investigate relations between the correlation of the signal and the noise and noise cancellation using NLMS algorithm. For this purpose, different configurations are shown by combining single-frequency sine wave, multifrequency sine wave, white noise, environment noise, and speech signal. The noise canceller circuit is described. NLMS algorithm is summarized. We apply spectrum analysis to the circuit to derive two basic relationships for evaluating the power spectrum of the canceller output in two cases:
First Case: UNCORRELATED SIGNAL AND NOISE.
Second Case: CORRELATED SIGNAL AND NOISE.

A data bank showing the relation between noise canceller performance and the signal-noise cross-correlation is established. This table is analysed, in order to see the conformity between the simulation results and the theory derived previously.

Then we discuss in the summary the confrontation of the theory and the simulation results.

2 NOISE CANCELLER CIRCUIT DESCRIPTION

The noise canceller circuit is composed of two second-order IIR filters. The filter

'F1' and the filter 'F2' represent the transfer function of the path of the noise, and its replica, respectively. A FIR transversal adaptive filter with 20 tap-weights is employed. Two adders are used to form the desired response and the output of canceller. The inputs of the filter 1 and the filter 2 are generated from the same noise source. The adaptive filter tap-weights-vector is adjusted based on the normalized least mean square(NLMS) algorithm which is summarized bellow.

Summary of normalized LMS algorithm [2]

$$Initialization : W(0) = 0$$

$$y(n) = W_n^T U_n$$

$$e(n) = d(n) - y(n)$$

$$\mu(n) = \frac{2\tilde{\mu}}{a + \sum_{k=1}^{M} u^2(n-k+1)}$$

$$W(n+1) = W(n) + \mu(n)U(n)e(n)$$

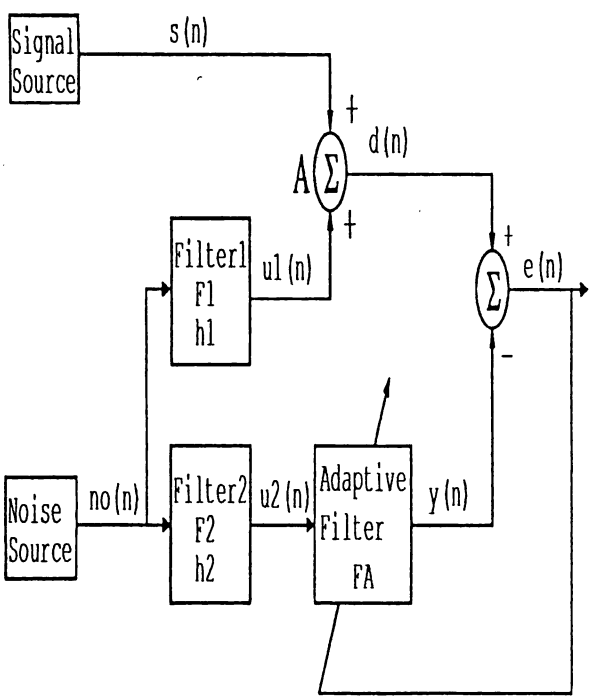

Figure 1: Noise canceller circuit

Where, [0] denotes the null vector , $\mu(n)$ the time varying step-size parameter, M the adaptive filter length, a is a small positive constant and the dimensionless step-size $0 < \tilde{\mu} \leq 1$

3 THEORETICAL ANALYSIS

In order to predict the simulation result to some extent, theoretical analysis is carried out, by applying spectrum analysis, to the noise canceller circuit. By doing so, mathematical formulas, are derived. The adaptive filter is assumed to operate in a stationary environment.

First Step

Relation giving S_{u_2d}.

The canceller mean square error(MSE) is given by:

$$\epsilon^2 = R_{d,d}(0) - 2\sum_{k=1}^{M} w(k)R_{u_2d}(k-1) + \sum_{k=1}^{M}\sum_{p=1}^{M} w(k)w(p)R_{u_2u_2}(k-p) \quad (1)$$

Where, $R_{d,d}(0)$ is the autocorrelation function of the desired response at lag zero, $R_{u_2d}(k-1)$ the cross-correlation function of the adaptive filter input and the desired response at lag $(k-1)$, $R_{u_2u_2}(k-p)$ the autocorrelation function of the adaptive filter input at lag $(k-p)$, and $w(k)$ the adaptive filter tap-weight of index k, M its length.

The adaptive filter convergence is assumed when the gradient of the MSE is null.

$$\frac{\partial \epsilon^2}{\partial w(k)} = 0 \quad (2)$$

By solving (2) and applying DFT(Discret Fourier Transform), we obtain.

$$S_{u_2d}(l) = F_A(l)S_{u_2u_2}(l) \quad (3)$$

Where, S_{u_2d}, F_A, and $S_{u_2u_2}$ are the cross-spectrum of the adaptive filter input and the desired response, the adaptive filter frequency response after convergence, and the adaptive

TABLE I: Noise canceller data bank:NLMS Algorithm

Combination / SNR1	Voice1/ Multitone	White noise/ White noise	Voice1/Work station noise	Voice1/ Voice2	Multitone/White noise+ one common frequency
-20 dB	MXR=0.0010 SNR2=22 Delta=42	MXR=0.0018 SNR2=18 Delta=38	MXR=0.0021 SNR2=17 Delta=37	MXR=0.0031 SNR2=9 Delta=29	MXR=0.0287 SNR2=4 Delta=24
-10 dB	MXR=0.0032 SNR2=22 Delta=32	MXR=0.0057 SNR2=19 Delta=29	MXR=0.0067 SNR2=17 Delta=27	MXR=0.0098 SNR2=9 Delta=19	MXR=0.0906 SNR2=4 Delta=14
0 dB	MXR=0.0100 SNR2=24 Delta=24	MXR=0.0179 SNR2=21 Delta=21	MXR=0.0210 SNR2=20 Delta=20	MXR=0.0310 SNR2=10 Delta=10	MXR=0.2865 SNR2=4 DELTA=4
+10 dB	MXR=0.0317 SNR2=28 Delta=18	MXR=0.0565 SNR2=27 Delta=17	MXR=0.0663 SNR2=23 Delta=13	MXR=0.0981 SNR2=10 Delta=0	MXR=0.9061 SNR2=4 Delta=-6
+20 dB	MXR=0.1001 SNR2=30 Delta=10	MXR=0.1786 SNR2=29 Delta=9	MXR=0.2096 SNR2=24 Delta=4	MXR=0.3102 SNR2=9 Delta=-11	MXR=2.8653 SNR2=4 Delta=-16

SNR1:Signal-To-Noise ratio before cancellation SNR2:Signal-To- noise ratio after cancellation

MXR:Croos-correlation mean Delta=SNR2-SNR1

filter input autospectrum, respectively. l is the discret frequency index.

Second Step

The canceller output power spectrum is given by:

$$S_{e,e}(l) = S_{d,d}(l) - F_A(l)S_{d,u_2}(l) \qquad (4)$$

Where, S_{d,u_2}, is the cross-spectrum of the desired response and the adaptive filter input. By substituting (3) in (4), we obtain

$$S_{e,e}(l) = S_{d,d}(l) - \frac{|S_{u_2d}(l)|^2}{S_{u_2u_2}(l)} \qquad (5)$$

Where $S_{d,d}$ is the autospectrum of the desired response.

Third Step

Relation giving $S_{d,d}$, and explicit expression of S_{u_2d} in the following two cases.

Case 1: UNCORRELATED SIGNAL AND NOISE

$$S_{d,d}(l) = S_{s,s}(l) + |F_1(l)|^2 S_{no,no}(l) \qquad (6)$$

$$S_{u_2d}(l) = F_1(l)F_2(l)S_{no,no}(l) \qquad (7)$$

Case 2: CORRELATED SIGNAL AND NOISE

$$\begin{aligned} S_{d,d}(l) &= S_{s,s}(l) + |F_1(l)|^2 S_{no,no}(l) \\ &\quad + F_1(l)S_{s,no}(l) + F_1^*(l)S_{s,no}^*(l) \end{aligned} \qquad (8)$$

$$S_{u_2d}(l) = S_{s,no}^*(l)F_2(l) + F_1(l)F_2(l)S_{no,no}(l) \qquad (9)$$

Where, $S_{s,s}$, $S_{no,no}$ and $S_{s,no}$ are the autospectrum of the signal, the autospectrum of the noise, the signal and noise cross-spectrum, respectively. [*] denotes the complex conjugate operator.

Fourth Step

Final expression of $S_{e,e}$ in the two cases. It is well known that: $S_{u_2u_2}(l) = |F_2(l)|^2 S_{no,no}(l)$

Case 1: UNCORRELATED SIGNAL AND NOISE

We combine (5), (6), and (7), and obtain.

$$S_{e,e}(l) = S_{s,s}(l) \qquad (10)$$

Case 2: CORRELATED SIGNAL AND NOISE

We combine (5), (8), and (9), and obtain.

$$S_{e,e}(l) = S_{s,s}(l) - \frac{|S_{s,no}(l)|^2}{S_{no,no}(l)} \qquad (11)$$

From (10) and (11), we make two deductions.
1. When the signal and the noise are noncorrelated, the canceller output is free of noise.
2. When the signal and the noise are correlated, the cross-spectrum produces the cancellation of some signal frequency components; that is, its distortion. Furthemore, we can notice that equation 10 is a special case of equation 11.

4 COMPUTER SIMULATION

Computer simulation is carried out. The following five combinations are taken into account: (1) Voice1/Multitone, (2) White noise/White noise, (3) Voice1/Workstation noise, (4) Voice1/Voice2, and then, (5) Multitone/White noise + one commun frequency. For combination 2, the signal and the noise have defferent seed. For combination 5 the signal is a 3 frequency-sine wave. A data bank shown by table 1 establishes the relation between noise canceller performance and the signal-noise cross-correlation. The cross-correlation is calculated by everaging the absolute value of the cross-correlation function. The noise level is normalized with variance equals to unity all over table 1. The signal is scaled in order to meet the adequate signal-to- noise ratio. The signal-to-noise ratio is evaluated at the generation sources. The signal-to-noise ratio after convergence is evaluated by considering the last 2000 samples of the canceller output and its residual noise. All the signals have uniform length equals to 20000. Fig. 2. and 3 illustrate the simulation results.

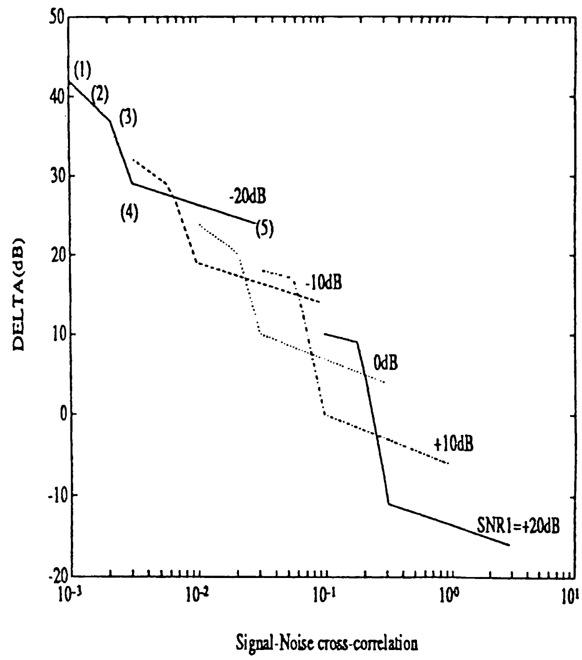

Figure 2: Cross-correlation and improvement relation

SNR1: Signal-to-noise ratio before cancellation

SNR2: Signal-to-noise ratio after cancellation

DELTA=SNR2-SNR1: Improvement

Horizontal axis: Signal noise cross correlation (semilogarithmic scale)

Vertical axis: SNR2, DELTA(linear scale)

These figures show that noise canceller performance is inversely proportional to the signal-noise cross-correlation. Hence, it has been proven that the theoretical analysis is well

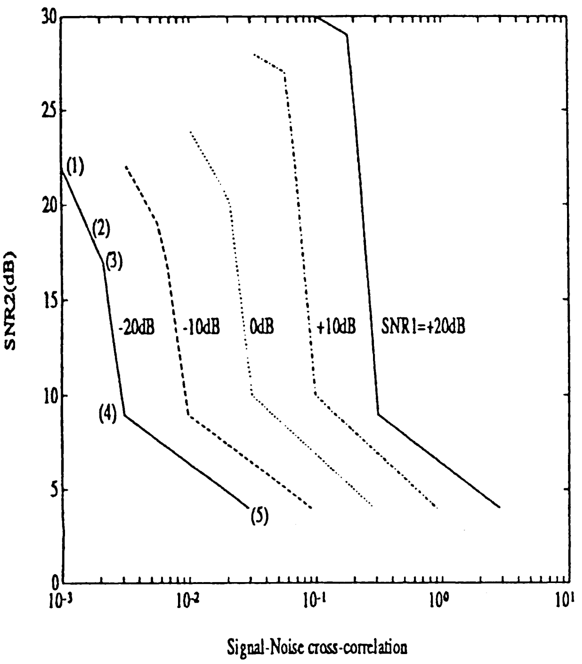

Figure 3: Cross-correlation and SNR2 relation

supported by the simulation results as expected.

5 SUMMARY

Noise cancellation has been analysed from a mathematical point of view by deriving two fundamental relationships, and through computer simulation. When the signal and the noise are noncorrelated the cancellation is achieved perfectly. On the other hand of correlated signal and noise, the noise is cancelled, but at the same time, the signal is distorted. The coherence of the theoretical analysis has been demonstrated by the simulation results.

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