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Optimum Order Assignment on Numerator and Denominator for IIR Adaptive Filters and An Approach toward Automation of Assignment Process

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ABSTRACT

This paper investigates effects of order assignment(OA) on the numerator and the denominator of a separate form of IIR adaptive filters. The filter coefficients are adjusted using the equation error(EE). The system identification error is approximately proportional to the EE as the OA approaches toward an optimum one. Next, an automatic OA method is proposed. The OA is gradually adjusted so as to minimize the EE. The optimum OA can be tracked for time varying systems. It is also demonstrated through simulation that the IIR adaptive filters can escape from the unstable state.

INTRODUCTION

For system identification problems, such as noise and echo cancellation, FIR adaptive filters are mainly used for their simple adaptation and numerical stability. When the unknown system is a high-Q resonant system, having a very long impulse response, IIR adaptive filters are more efficient for reduction in the order of a transfer function.

It is to be noted here that a number of IIR or IIR like methods have been proposed both in adaptive signal processing and system identification community [1]-[4]. However, the class IIR itself comes off a more general ARMAX model family[5]. IIR filters can be classified broadly into two groups according to the error criteria, used for adaptation. One is the equation error and the other is the output error. Though it has been shown that these two forms of error are equivalent in the sense that any of them can be used as the minimization criterion [6], the output error corresponds to actual transfer function error, whereas the equation error is a filtered version of it [7][8][9]. One way to realize the equation error IIR adaptive filter is a separate form, in which the numerator and the denominator are separately realized and adjusted.

There exist some methods to estimate optimum order for non adaptive system identification problems; one of them is Akaike's Information Criterion (AIC) [10], which uses the final prediction error to calculate optimum order in least square problems. Other methods, also have been proposed such as in speech processing problems, [11][12][13]. However, it seems that investigations around the "separated realization of the IIR" adaptive filters, are rare. Methods for estimating optimum tap assignment for a given number of total taps may not be easy available for this particular structure of the IIR filter.

However, in the actual applications of an IIR adaptive filter, the order of the unknown system is not known. In this case, it is very important to estimate the total order and the order assignment on the numerator and the denominator. Especially, when the total number of coefficients is limited, performance of the IIR adaptive filter is very sensitive to this order assignment. Generally, the total order of an practical adaptive filter is fixed but it is possible to distribute the number of orders among the the numerator and the the denominator of IIR adaptive filters.

In this paper, effects of order assignment in the separate realization of the IIR adaptive filter is investigated. Furthermore, efficiency of the equation error to evaluate the performance of the filters is investigated. Also, the stability problem in a process of finding the optimum order assignment is discussed. Finally, a method for automatic tap assignment is shown. Recursive least square(RLS)[14] algorithm is employed. The system identification problem is taken into account.

SEPARATE REALIZATION OF IIR ADAPTIVE FILTER

Network Structure

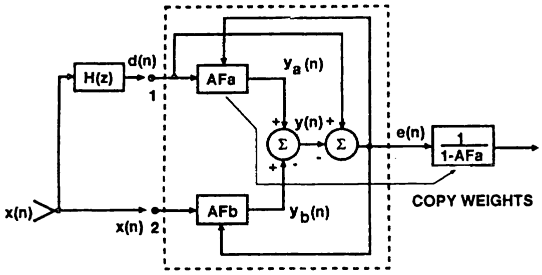

Fig. 1 Separate realization of IIR adaptive filter.

A block diagram of the separate realization of the IIR adaptive filter is shown in Fig.1. H(z) indicates a transfer function of the unknown system to be identified. $AF_a(z)$ and $AF_b(z)$ construct the denominator and numerator, respectively. x(n) is the far end signal or noise for example. The output of H(z), denoted d(n), is used as the desired response. d(n) should be canceled out. Furthermore, the near-end signal s(n) is added at the terminal 1. A transfer function for s(n) should be unity. For this purpose, an all pole filter, having a transfer function $[1 - AFa(z)]^{-1}$ is used after the equation error. The adjusted weights are copied to this block.

Equation Error Evaluation

From the Fig.1 we see that

$$y(n) = y_a(n) + y_b(n) \tag{1}$$
$$e(n) = d(n) - y(n) \tag{2}$$

The error given by Eq. (2) is the equation error. The z-transform of this error is derived in the following. Letting $D(z)$, $X(z)$, $Y_a(z)$, $Y_b(z)$, $Y(z)$, and $E(z)$ be z-transform of $d(n)$, $x(n)$, $y_a(n)$, $y_b(n)$, $y(n)$, and $e(n)$, respectively, we obtain

$$D(z) = H(z)X(z) \tag{3a}$$
$$Y_a(z) = AF_a(z)D(z) \tag{3b}$$
$$Y_b(z) = AF_b(z)X(z) \tag{3c}$$
$$Y(z) = Y_a(z) + Y_b(z) \tag{3d}$$
$$E(z) = D(z) - Y(z) \tag{3e}$$

By eliminating $D(z)$, $Y_a(z)$ and $Y_b(z)$, $E(z)$ can be expressed as

$$E(z) = [H(z) - H(z)AF_a(z) - AF_b(z)]X(z) \tag{4}$$

The ideal solution can be obtained by setting the inside of the bracket to be zero.

$$H(z) - H(z)AF_a(z) - AF_b(z) = 0 \tag{5}$$

From this condition, the following relation is obtained.

$$H(z) = \frac{AF_b(z)}{1 - AF_a(z)} \tag{6}$$

Let F(z) and G(z) be used to express the numerator and the denominator of H(z), respectively. Equation (14) shows $AF_a(z)$ and $AF_b(z)$ correspond to G(z) and F(z), respectively. Therefore, if $AF_a(z)$ and $AF_b(z)$ have the same order as that of G(z) and F(z), respectively, then we can say a unique ideal solution can exist.

On the other hand, if the condition expressed in Eq.(5) cannot be satisfied, $H(z)$ should include an error term $\Delta H(z)$ in Eq.(12), and their relation is rewritten as

$$H(z) + \Delta H(z) = \frac{AF_b(z) + E(z)/X(z)}{1 - AF_a(z)} \tag{7}$$
$$\Delta H(z) = \frac{E(z)/X(z)}{1 - AFa(z)} \tag{8}$$

The above equation shows that the equation error $E(z)$ is weighted by $X(z)$ and $1 - AF_a(z)$ in the transfer function error criterion. In brief, the outcome of this section is: Equation(8) indicates that 'equation error' can not exactly represent the transfer function error. This raises the following question: Is it possible to obtain the global minimum solution by using the equation error? This problem will be discussed in the later section.

Error Criteria

The equation error is evaluated in this paper using the following relation.

$$E_{eq} = \frac{1}{K} \sum_{i=n_0}^{n_0+K-1} | e(i) |^2 \tag{9}$$

It is assumed that at $n = n_0$ the adaptation already converges. K is the number of error sample taken into account. As discussed in the previous section that E_{eq} does not directly correspond to the transfer function error, therefore, in simulation, efficiency of the equation error is evaluated by comparing the following error criteria.

$$E_{imp} = 10log_{10}\frac{\| \mathbf{h} - \mathbf{h}_{AF} \|^2}{\| \mathbf{h} \|^2} \tag{10}$$
$$\mathbf{h} = [h(0), h(1), ..., h(L-1)]^T \tag{11}$$
$$\mathbf{h}_{AF} = [h_{AF}(0), h_{AF}(1), ..., h_{AF}(L-1)]^T \tag{12}$$

$\| \cdot \|$ indicates Euclidean norm. $h_H(n)$ and $h_{AF}(n)$ are impulse responses of H(z) and $H_{AF}(z)$ shown below, respectively.

$$H_{AF}(z) = \frac{AF_b(z)}{1 - AF_a(z)} \tag{13}$$

respectively.

AUTOMATIC ORDER ASSIGNMENT

A block diagram of automatic order assignment is shown in Fig.2. This diagram is based on the basic diagram of Fig.1. As shown in Fig.2, two adaptive IIR filters, AF_{main} and AF_{aux} are used in the structure. Both the adaptive filters are adapted simultaneously using the respective equation errors. Again it is to be noted that

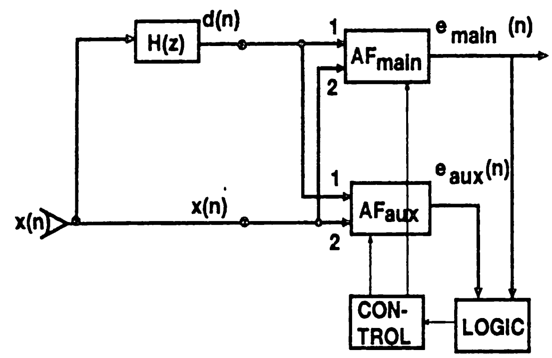

Fig. 2 Block diagram of automatic tap assignment problem.

the input x(n) and the desired signal d(n) is applied to both the AF_{main} and AF_{aux} at the same time. The logic block receives the respective errors and does some mathematical and logic operations such as squaring, doing cumulative sums and comparing etc. The logic block is connected with the control block which performs operations such as copy the tap values, increase or decrease the tap ratio N/D at the end of some interval.

The operation of the diagram is clarified by the following algorithm. N and D indicate the number of taps in the numerator and the denominator, respectively. In this paper, the total number of taps, that is N+D, is assumed to be fixed. $\frac{N_{main}}{D_{main}}$ and $\frac{N_{aux}}{D_{aux}}$ indicate tap ratios, and W_{main} and W_{aux} mean the coefficient vectors for the main and auxiliary adaptive filters, respectively.

The operation expressed in the following algorithm is always continued. An example, showing relation between two tap ratio movings, is illustrated in Fig.3. Solid lines and dashed lines indicate $\frac{N_{main}}{D_{main}}$ and $\frac{N_{aux}}{D_{aux}}$, respectively. It is assumed that the tap ratio closed to the optimum can provide small error. When the tap ratios approach to the optimum, $\frac{N_{aux}}{D_{aux}}$ precede that of the main filter. After $\frac{N_{main}}{D_{main}}$ reaches the optimum, $\frac{N_{aux}}{D_{aux}}$ slightly vibrates around the optimum.

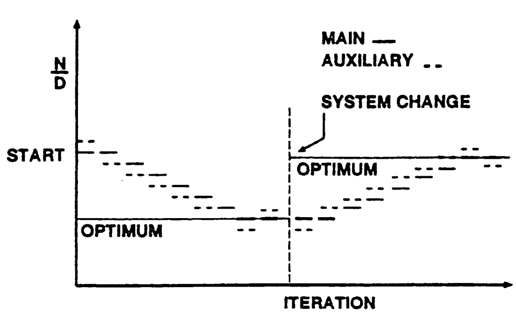

Fig. 3 Example of adjusting tap ratios.

Step 0 Determine the total number of taps N+D, which is fixed in the following steps.

Step 1 Pre-assign a tap ratio $\frac{N_{main}(0)}{D_{main}(0)}$ to the main adaptive filter AF_{main}, where $N_{main}(0) \approx D_{main}(0)$.
Also pre-assign the tap ratio $\frac{N_{aux}(0)}{D_{aux}(0)}$ to the auxiliary adaptive filter.
$N_{aux}(0) = N_{main}(0) + 1$
$D_{aux}(0) = D_{main}(0) - 1$
Define direction parameter δ=1
Initialize both adaptive filters with zero initial tap values:
$W_{main}(0) = 0$
$W_{aux}(0) = 0$

Step 2 Adapt both adaptive filters simultaneously for some iterations.

Step 3 Calculate $E_{main}(k)$ and $E_{aux}(k)$, cumulative sums of errors $e^2_{main}(n)$ and $e^2_{aux}(n)$, for corresponding adaptive filters, in the kth interval.

Step 4 If $E_{aux}(k) < E_{main}(k)$, go to Step 5, else go to Step 6.

Step 5 Set the tap ratio and the coefficients of the adaptive filters such that :

Main adaptive filter:
$N_{main}(k+1) = N_{aux}(k)$
$D_{main}(k+1) = D_{aux}(k)$
$W_{main}(k+1) = W_{aux}(k)$
Auxiliary adaptive filter:
$N_{aux}(k+1) = N_{aux}(k) + \delta,$
$D_{aux}(k+1) = D_{aux}(k) - \delta,$
$W_{aux}(k+1) = W^*_{aux}(k)$
Go to Step2

Step 6 Set the tap ratio and the coefficients of the adaptive filters such that:

Main adaptive filter:
$N_{main}(k+1) = N_{main}(k)$
$D_{main}(k+1) = D_{main}(k)$
$W_{main}(k+1) = W_{main}(k)$
Auxiliary adaptive filter:
$N_{aux}(k+1) = N_{aux}(k+1) - \delta$
$D_{aux}(k+1) = D_{aux}(k+1) + \delta$
$W_{aux}(k+1) = W^*_{aux}(k)$
$\delta = -\delta$
Go to Step 2.

$W^*_{aux}(k)$ means the following. In $AF_{aux}(z)$ at the kth interval, the highest order term is removed, or the highest order term is added with zero coefficients in the numerator or denominator polynomial of z^{-1}.

SIMULATION RESULTS

Unknown System and Input Signal

The following transfer function is used for the unknown system.

$$H(z) = \frac{1 + 1.5z^{-1} + 0.9025z^{-2}}{D(z)} \quad (14)$$

$$D(z) = [1 - 0.9z^{-1} + 0.8z^{-2} - 0.7z^{-3} + 0.6z^{-4} -0.5z^{-5} + 0.4z^{-6} - 0.3z^{-7} + 0.2z^{-8} + 0.1z^{-9}]$$

As shown in Fig.4, the unknown system has 9 poles and 2 zeros. The sampling frequency is set to 2 Hz. The input signal is a white noise. RLS algorithm is used as the adaptation mechanism.

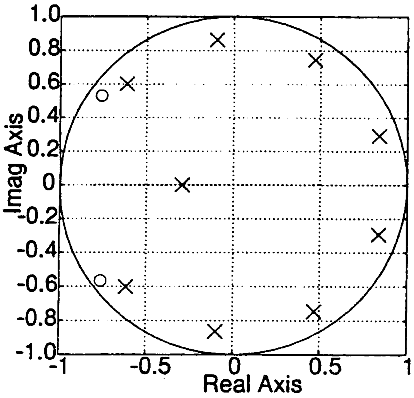

Fig.4 Pole-Zero location of the unknown system. × indicates pole and ○ indicates zero

Effects of Order Assignment

From Eq.(14), it can be noted that total order of the unknown system is 11th order, and the total number of coefficients is 13. By limiting these number to be invariant, effects of order assignment on the error criteria described in the previous section are investigated.

Table 1 shows the simulation results. N/D means ratio of the number of taps of the numerator and the denominator. Adaptation was carried out independently for each ratio starting from zero initial coefficient.
The optimum ratio is 3/10.

From Table 1, it can be noted that the equation error E_{eq} can monotonously decrease toward the optimum assignment. Around optimum assignment, E_{eq} is proportional to the other error criteria, E_{imp}. From these results, it can be concluded that the equation error as described in Eq.(9), can be used in searching for the optimum order assignment.

Stability Problem

Table 1 includes one problem, that is instability, which can be detected by investigating E_{imp}. The impulse response of the adaptive filter diverges when poles locate outside the unit circle. For examples, in the cases of N/D=5/8 and 2/11, the filter falls into the unstable state.

One way to recover from this instability is to reciprocate unstable roots inside the unit circle [15] at every iteration. In this paper, however, the stability problem

Table 1 Error criteria for different tap ratios without measurement noise and using 500 data samples.

N/D	E_{eq} $\times 10^{-4}$	E_{imp} (dB)
12/1	31.82	-15.23
11/2	52.03	-14.47
10/3	52.0	-10.46
9/4	41.0	-8.97
8/5	33.0	-9.05
7/6	33.0	-9.13
6/7	32.0	-11.57
5/8	27.0	UNSTABLE
4/9	1.50	-33.02
3/10	**0.87**	**-43.86**
2/11	30.0	UNSTABLE
1/12	2.95	-21.73

is further investigated in the following condition. The ratio N/D initially starts from 5/8, and is successively changed every 500 iterations toward 2/11. 500 iterations has been determined in order to guarantee convergence by experience. The adjusted coefficients for N/D=n/m are used as the initial guess for N/D=(n-1)/(m+1). Simulation results are shown in Table 2.

Table 2 Stability analysis in adaptation process. Ratio N/D is changed from 5/8 to 2/11. Change of ratio occurs after every 500 iterations.

N/D	5/8	4/9	3/10	2/11
E_{eq}	27.0×10^{-4}	3.28×10^{-4}	0.59×10^{-4}	4.84×10^{-4}
E_{imp}	Unstable →	Stable -23.11 dB	Stable -42.05 dB	Stable -27.88 dB
E_{amp}	-22.07 dB	-22.57 dB	-43.07 dB	-31.28 dB
Iteration	0 ~ 500	501 ~ 1000	1001 ~ 1500	1501 ~ 2000

The resulting error criteria are a little different from those in Table 1, because the initial guess of the coefficients are different. The filter falls into the unstable state in the ratio of 5/8. However, it can recover from the unstable state in the following adaptation using the ratios toward the optimum. In Table 1, the result is unstable for N/D=2/11. However, in Table 3, it can be stable. This property of the equation error can guarantee the possibility to find the optimum order assignment in stable state in an adaptive filter. This is an important outcome of this investigation.

On the contrary, the direct form(output error) method can not continue adaptation after the IIR filter falls into unstable state, because the error diverges and hence can not be used for adaptation. Furthermore, if adaptation starts from zero initial condition, the filter also falls into unstable state like N/D=2/11 in Table 1.

Automatic Order Assignment

Table 1 and Table 2 show a good possibility of automatic order assignment. Table 2 shows that order assignment is changed from 5/8 to 2/11 during a total of 2000 iterations. E_{eq} is the smallest at 3/10 ratio, which is equal to the true unknown system. Simulation results concerning automatic tap assignment are shown in Table 3. However, this table shows the simulation results which does not counter tracking problem and and all pre-stored tap ratios are checked for comparison. When the tap ratio is changed to a higher or lower ratio, previous tap values are copied with the addition of a zero or removal of the last element, whatever, appropriate.

Table 3 Error for different tap ratios with -20 dB measurement noise and using a block length of 100.

ITER	N/D(MAIN)	E_{main}	N/D(AUX)	E_{aux}
100	12/1	0.6605	11/2	1.4204
200	12/1	0.6063	10/3	1.2826
300	12/1	0.4526	9/4	1.0294
400	12/1	0.4226	8/5	0.5065
500	12/1	0.3433	7/6	0.1900
600	7/6	0.0871	6/7	0.0729
700	6/7	0.0389	5/8	0.0285
800	5/8	0.0140	4/9	0.0047
900	4/9	0.0012	**3/10**	**0.0010**
1000	3/10	0.0008	2/11	0.0043
1100	3/10	0.0004	1/12	0.0067
1200	3/10	0.0001	1/12	0.0028
1300	3/10	0.0001	1/12	0.0013

The Table 3 shows the simulation results for the case, where, the block size of iteration is taken 100 and the desired signal is buried in a measurement noise of -20 dB compared to unity variance. The unknown system was kept unchanged. In the table, the tap ratio of AF_{main} and its corresponding error E_{main} are shown in second and third column, respectively. Similarly fourth and fifth columns show the tap ratio of AF_{aux} and corresponding error E_{aux}. The first column shows the iteration level. The fourth i.e., the N/D(AUX) column contains every possible order assignments. It may be seen from the column that the 9th entry of the column(counting from the top) corresponds to the minimum error. So,the 3/10 ratio is the optimum order assignment which agrees with the unknown system.

Figure 5 shows the plots of the simulation results of Table 3. Figures 5(a) and 5(b) shows the frequency and impulse responses of the adaptive filter AF_{main} after 1300 iterations and the unknown system. The solid and dotted curves represent the adaptive filter and the ideal curves, respectively. Figure 5(c) shows the equation error of the same adaptive filter. From Table 3, it may be noticed that all pre-stored tap ratios were checked automatically within 11 blocks i.e., 1100 iterations. However, the equation error in Fig.5(c) shows a total number of 1300 iterations. The figures clearly show that the identification is satisfactory.

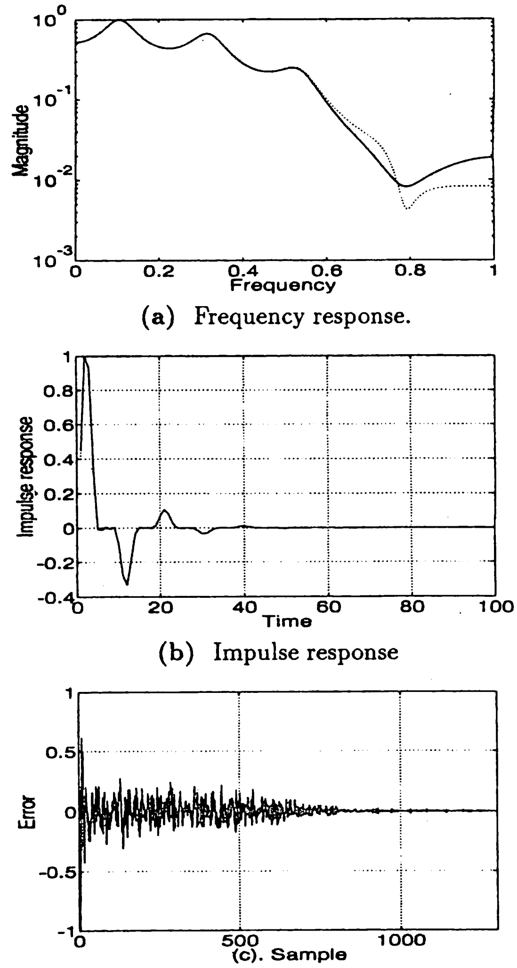

Figure 5 Simulation results for automatic tap assignment problem. All plots corresponds to AF_{main}. In (a) and (b), solid and dotted lines represent the adaptive filter after 1300 iterations and unknown system respectively.

CONCLUSION

The performance of the adaptive filter has been discussed based on the order assignment. Around the optimum order assignment, the equation error is approximately proportional to the transfer function error. In some cases, the filter may fall into unstable behavior. However, this adaptive filter can recover from the unstable state as its order assignment approaches toward the optimum. Therefore, it can be used for finding the optimum solution. An automatic assignment of order has been also proposed.

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