A Learning Algorithm for Blind Source Separation Mixing Feedforward and Feedback

Juanjuan Bian Kenji Nakayama Akihiro Hirano

Division of Electrical and Computer Engineering,
Graduate School of Natural Science & Technology, Kanazawa Univ.
nakayama@t.kanazawa-u.ac.jp

ABSTRACT

A feedback (FB-) Blind Source Separation (BSS) system has a good separation performance with no signal distortion. This approach, however, requires some delay condition in the mixing process. A new BSS approach, which mixing the FB process and a feedforward (FF) process, has been proposed. In this paper, we propose a learning algorithm for this FF-FB mixed BSS system. The input signals are applied to the FF-FB mixed BSS system and the output signals are calculated. By using the output signals, the filter coefficients of the fully recurrent (FB-) BSS system are updated based on the gradient descent algorithm. These filter coefficients are transferred into the filter coefficients of the FF-FB mixed BSS system. Simulation results by using the two channel FF-FB mixed BSS system and speech signals show good separation performance.

1. INTRODUCTION

Recently, many kinds of information are transmitted and processed. Signal processing including noise canc-elation, echo cancellation, equalization of transmission lines, restoration of signal have been becoming very important technology. In some cases, we do not have enough information about signal sources and interference. Furthermore, their mixing process and transmission are not well known in advance. Under these situations, blind source separation (BSS) system using statistical property of the signal sources has become important.

Jutten et all proposed a blind separation algorithm for a fully recurrent network based on statistical independence and symmetrical distribution of the signal sources [1]-[3]. Convlutive mixture models have been discussed [6]. However, in this fully recurrent BSS, in order to gain a good performance delay time must be imposed on the mixing process. To solve this problem a new structure FF-FB mixed system has been proposed [8].

In this paper, a learning algorithm is proposed for the FF-FB mixed BSS system. The input signals are applied to the FF-FB mixed BSS system and the output signals are calculated. By using the output signals, the filter coefficients of the fully recurrent (FB-) BSS system are updated based on the gradient descent algorithm. These filter coefficients are transferred into the filter coefficients of the FF-FB mixed BSS system. Simulation results by using the 2-channel BSS system and speech signals will be shown to confirm efficiency of the proposed method.

2. NETWORK AND SEPARATION CONDITIONS FOR FB-BSS SYSTEM

2.1. Network Structure

A fully recurrent separation model proposed by Jutten et all [1],[4] is shown in Fig.1. The signal sources s_i are mixed through linear combination resulting in x_j. Fig.2 shows the FIR filters structure used in the feedback circuits.

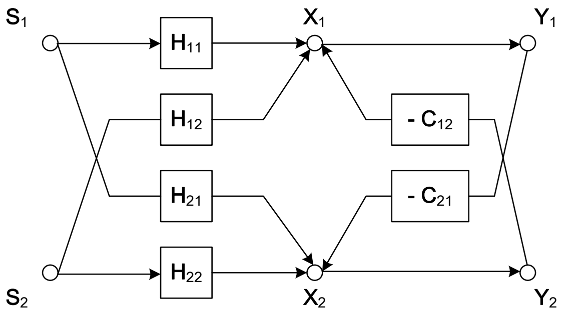

Fig.1: Block diagram of recurrent blind separation

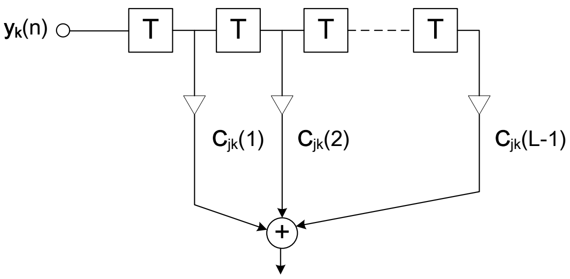

Fig.2: FB filter with first tap $c_{jk}(0) = 0$

In order to avoid a free delay loop, the coefficients of the 0th order term is set to be zero, that is $c_{jk}(0) = 0$.

2.2 Separation Conditions

The signal sources $s_i(n),\ i = 1,2,\cdots,N$ are combined through the unknown convolutive mixture block, which has the impulse response $h_{ji}(m)$, and are sensed at N points, resulting in $x_j(n)$.

$$x_j(n) = \sum_{i=1}^{N} \sum_{m=0}^{M_{ji}-1} h_{ji}(m) s_i(n-m) \tag{1}$$

The output of the unmixing block $y_j(n)$ is given by

$$y_j(n) = x_j(n) - \sum_{\substack{k=1 \\ \neq j}}^{N} \sum_{l=1}^{L_{jk}-1} c_{jk}(l) y_k(n-l) \tag{2}$$

Let's define the z-transform of $s_i(n), x_j(n)$ and $y_k(n)$ as $S_i(z), X_j(z), Y_k(z)$ and they are related as follows:

$$\boldsymbol{X}(z) = \boldsymbol{H}(z)\boldsymbol{S}(z) \tag{5}$$

$$\boldsymbol{Y}(z) = \boldsymbol{X}(z) - \boldsymbol{C}(z)\boldsymbol{Y}(z) \tag{6}$$

$$\boldsymbol{S}(z) = [S_1(z), S_2(z)]^T \tag{7}$$

$$\boldsymbol{X}(z) = [X_1(z), X_2(z)]^T \tag{8}$$

$$\boldsymbol{C}(z) = \begin{bmatrix} 0 & c_{12}(z) \\ c_{21}(z) & 0 \end{bmatrix} \tag{9}$$

$$\boldsymbol{Y}(z) = [Y_1(z), Y_2(z)]^T \tag{10}$$

From these expressions, a relation between the signal sources and the unmixing outputs becomes:

$$\begin{aligned} \boldsymbol{Y}(z) &= \left(\boldsymbol{I} + \boldsymbol{C}(z)\right)^{-1} \boldsymbol{X}(z) \\ &= \left(\boldsymbol{I} + \boldsymbol{C}(z)\right)^{-1} \boldsymbol{H}(z)\boldsymbol{S}(z) \end{aligned} \tag{11}$$

In order to evaluate separation performance, the following matrix is defined.

$$\boldsymbol{P}(z) = \left(\boldsymbol{I} + \boldsymbol{C}(z)\right)^{-1} \boldsymbol{H}(z) \tag{12}$$

If each row and column of $\boldsymbol{P}(z)$ has only a single non-zero element, the signal sources $s_i(n)$ are completely separated at the outputs $y_k(n)$. However, since equalization of $\boldsymbol{H}(z)$ is not guaranteed, the separated signals have the following form:

$$Y_j(z) = P_{jj}(z) S_j(z) \tag{13}$$

The solution for good separation is given by [6]

$$C_{21}(z) = \frac{H_{21}(z)}{H_{11}(z)} \quad C_{12}(z) = \frac{H_{12}(z)}{H_{22}(z)} \tag{14}$$

$$\mathrm{y}_1(n) = \boldsymbol{h}_{11}^T \boldsymbol{s}_1(n) \quad \mathrm{y}_2(n) = \boldsymbol{h}_{22}^T \boldsymbol{s}_2(n) \tag{15}$$

2.3 Learning Algorithm and Delay Constraints

The learning algorithms for the FB-BSS system have been proposed in [4],[6]. Especially, effects of the transmission delays in the mixing block has been well analyzed in [6],[7]. In FB-BSS system, in order to achieve good separation performance, the following delay constraints are required. The transmission delay of $H_{12}(z)$ and $H_{21}(z)$ should be larger than the delay of $H_{11}(z)$ and $H_{22}(z)$.

3. FF-FB MIXED BSS SYSTEM

To avoid the delay constraints required in the FB-BSS system described in Sec.2.3, a new BSS structure has been proposed [8], which is called 'FF-FB mixed BSS system' in this paper. A learning algorithm is proposed for this BSS system.

3.1. Transformation between FIR Filters in FB-BSS and FF-FB Mixed BSS

Not considering the first-tap of the FIR filters used in feedback, the partial coefficients vectors $\bar{\boldsymbol{c}}_{12}$ and $\bar{\boldsymbol{c}}_{21}$ are defined as Eqs.(16) and (17).

$$\bar{\boldsymbol{c}}_{12} = [c_{12}(1), c_{12}(2), \cdots, c_{12}(L_{12}-1)]^{\mathrm{T}} \quad (16)$$

$$\bar{\boldsymbol{c}}_{21} = [c_{21}(1), c_{21}(2), \cdots, c_{21}(L_{21}-1)]^{\mathrm{T}} \quad (17)$$

Like $\bar{\boldsymbol{c}}_{12}$ and $\bar{\boldsymbol{c}}_{21}$, the partial output vectors are defined as Eqs. (18) and (19)

$$\bar{\boldsymbol{y}}_1(n) = [y_1(n-1), y_1(n-2), \cdots, y_1(n-L_{21}+1)] \quad (18)$$

$$\bar{\boldsymbol{y}}_2(n) = [y_2(n-1), y_2(n-2), \cdots, y_2(n-L_{12}+1)] \quad (19)$$

By using the past values $\bar{\boldsymbol{y}}_1(n)$, $\bar{\boldsymbol{y}}_2(n)$ and partial filters $\bar{\boldsymbol{c}}_{12}$ and $\bar{\boldsymbol{c}}_{21}$, let's define $u_1(n)$ and $u_2(n)$ as

$$u_1(n) = \bar{\boldsymbol{c}}_{12}^{\mathrm{T}} \bar{\boldsymbol{y}}_2(n) = \sum_{l=1}^{L_{12}} c_{12}(l) y_2(n-l) \quad (20)$$

$$u_2(n) = \bar{\boldsymbol{c}}_{21}^{\mathrm{T}} \bar{\boldsymbol{y}}_1(n) = \sum_{l=1}^{L_{21}} c_{21}(l) y_1(n-l) \quad (21)$$

Divided into two parts by the distinction of past-signal and current-signal, the output signals $y_1(n)$, $y_2(n)$ can be written in vector by

$$\boldsymbol{y}(n) = \begin{bmatrix} x_1(n) - u_1(n) \\ x_2(n) - u_2(n) \end{bmatrix} - \begin{bmatrix} c_{12}(0) y_2(n) \\ c_{21}(0) y_1(n) \end{bmatrix} \quad (22)$$

$$= \begin{bmatrix} x_1(n) - u_1(n) \\ x_2(n) - u_2(n) \end{bmatrix} - \begin{bmatrix} 0 & c_{12}(0) \\ c_{21}(0) & 0 \end{bmatrix} \begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix}$$

$$\boldsymbol{y}(n) = \begin{bmatrix} 1 & c_{12}(0) \\ c_{21}(0) & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1(n) - u_1(n) \\ x_2(n) - u_2(n) \end{bmatrix} \quad (23)$$

Equation (36) is further rewritten as follows:

$$\boldsymbol{y}(n) = \boldsymbol{V} \begin{bmatrix} x_1(n) - u_1(n) \\ x_2(n) - u_2(n) \end{bmatrix}$$

$$= \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} x_1(n) - u_1(n) \\ x_2(n) - u_2(n) \end{bmatrix} \quad (24)$$

where

$$\boldsymbol{V} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} 1 & c_{12}(0) \\ c_{21}(0) & 1 \end{bmatrix}^{-1} \quad (25)$$

Equations (20), (21) and (24) can be realized by the realized by the block diagram shown in Fig.3.

3.2. Network Structure of FF-FB Mixed BSS System

Figure 3 shows a new separation structure mixing the FB process and a feedforward process (FF-FB) according to Eq.(24).

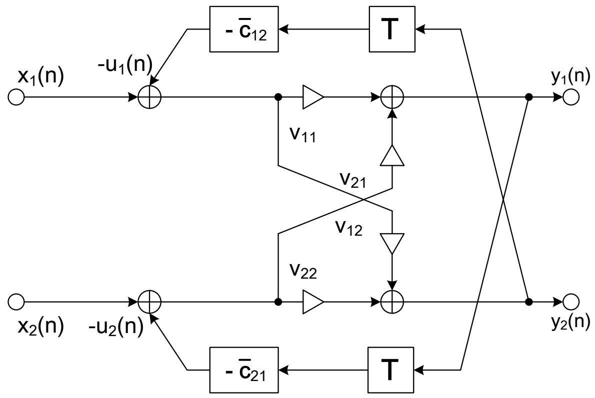

Fig.3: FF-FB mixed BSS system

$c_{12}(0)$ and $c_{21}(0)$ of the FIR filters in the FB-BSS are related to v_{11}, v_{12}, v_{21} and v_{22} in the FF-FB mixed BSS trough Eq.(25). Furthermore, $c_{12}(l)$ and $c_{21}(l)$, $l = 1, 2, \cdots, L_{12}$ or L_{21}, are directly related to the coefficients in c_{12}\{bar\} and c_{21}\{bar\}.

4. LEARNING ALGORITHM FOR FF-FB MIXED BSS SYSTEM

4.1. Basic Idea

As shown in Sec.3, the FB-BSS having the FIR filters in the feedbacks, which have the 0^{th} order coefficients $c_{12}(0)$ and $c_{21}(0)$, as shown in Fig.4 is equivalent to the FF-FB mixed BSS. The learning algorithms are well established in the FB-BSS [4], [6]. Therefore, we propose the following learning algorithm.

Step 1: Set the initial filter coefficients. $c_{12}(l)=0$, $c_{21}(l)=0$. From Eq.(25), $v_{11}=v_{22}=1$, $v_{12}=v_{21}=0$. All coefficients in c_{12}\{bar\} and c_{21}\{bar\} are zero.

Step 2: Apply the input signals $x_1(n)$ and $x_2(n)$ to the FF-FB mixed BSS system shown in Fig.3 and calculate the output signals $y_1(n)$ and $y_2(n)$.

Step 3: By using $y_1(n)$ and $y_2(n)$, update the FIR filter coefficients, $c_{12}(l)$ and $c_{21}(l)$, $l=1,2,\cdots,L_{12}$ and L_{21} shown in Fig.4, following the gradient descent algorithm [6].

Step 4: Transfer $c_{12}(0)$ and $c_{21}(0)$ to v_{11}, v_{12}, v_{21}, v_{22} through Eq.(25). Directly use $c_{12}(l)$ and $c_{21}(l)$, $l=1,2,\cdots,L_{12}$ and L_{21} ,as the coefficients in c_{12}\{bar\} and c_{21}\{bar\} in the FF-FB mixed BSS.

Step 5: Return to Step 2.

Step 2 through Step 5 are repeated sample by sample. In the following sub-sections, the gradient descent algorithm is described in details [6].

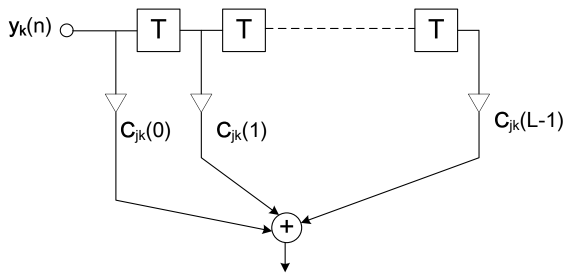

Fig.4: FIR filter in FB-BSS system used for updating

4.2. Cost Function

From Eq.(11), the outputs can be expressed for 2-channel blind separation as follows:

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \frac{1}{1-C_{12}(z)C_{21}(z)} \begin{bmatrix} 1 & -C_{12}(z) \\ -C_{21}(z) & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} H_{11}(z) & H_{12}(z) \\ H_{21}(z) & H_{22}(z) \end{bmatrix} \begin{bmatrix} S_1(z) \\ S_2(z) \end{bmatrix} \tag{26}$$

$$= \frac{1}{1-C_{12}(z)C_{21}(z)} \times$$

$$\begin{bmatrix} H_{11}(z)-C_{12}(z)H_{21}(z) & H_{12}(z)-C_{12}(z)H_{22}(z) \\ H_{21}(z)-C_{21}(z)H_{11}(z) & H_{22}(z)-C_{21}(z)H_{12}(z) \end{bmatrix} \begin{bmatrix} S_1(z) \\ S_2(z) \end{bmatrix} \tag{27}$$

The diagonal elements of Eq.(27) cannot be zero and the non-diagonal elements can be equal to zero by adjusting the feedback coefficients $C_{12}(z)$ and $C_{21}(z)$ that a cost function can be defined as follows [6]

$$J_j(n) = E\left[q(y_j(n))\right] \tag{28}$$

$q(\,)$ is an even function with a single minimum point. By minimizing this cost function, $C_{12}(z)$and $C_{21}(z)$ can approach to Eq.(14). Referring to LMS algorithm for adaptive filters [5], $E\left[q(y_j(n))\right]$ can be replaced by the instantaneous value $q(y_j(n))$.

$$\hat{J}_j(n) = q\left(y_j(n)\right) \tag{29}$$

4.3. Update Equation for $\mathbf{C_{jk}(z)}$

The gradient of $\hat{J}_j(n)$ becomes

$$\frac{\partial \hat{J}_j(n)}{\partial c_{jk}(l)} = \frac{\partial q\left(y_j(n)\right)}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial c_{jk}(l)} \tag{30}$$

If $k=1$, then $j=2$, and vice versa.

$$y_j(n) = x_j(n) - \sum_{l=0}^{L_{jk}} c_{jk}(l) y_k(n-l)$$

$$= x_j(n) - c_{jk}(0)y_k(n) - \sum_{l=1}^{L_{jk}} c_{jk}(l) y_k(n-l)$$

$$= x_j(n) - c_{jk}(0)\left[x_k(n) - \sum_{l=0}^{L_{kj}} c_{kj}(l) y_j(n-l)\right]$$

$$- \sum_{l=1}^{L_{jk}} c_{jk}(l) y_k(n-l)$$

$$= x_j(n) - c_{jk}(0)[\ x_k(n) - c_{kj}(0)y_j(n)$$

$$- \sum_{l=1}^{L_{kj}} c_{kj}(l) y_j(n-l)] - \sum_{l=1}^{L_{jk}} c_{jk}(l) y_k(n-l) \tag{31}$$

Here, $u_k(n)$ is defined as

$$u_k(n) = \sum_{l=1}^{L_{kj}} c_{kj}(l) y_j(n-l) \tag{32}$$

According to Eqs.(31) and (32)

$$[1-c_{jk}(0)c_{kj}(0)]y_j(n) = x_j(n) - c_{jk}(0)[x_k(n) - u_k(n)] - u_j(n) \tag{33}$$

$y_j(n)$ can be given by

$$y_j(n) = \frac{x_j(n) - c_{jk}(0)[x_k(n) - u_k(n)] - u_j(n)}{1 - c_{jk}(0)c_{kj}(0)} \quad (34)$$

The partial of $y_j(n)$ by $c_{jk}(l)$ is

$$\frac{\partial y_j(n)}{\partial c_{jk}(0)} = -[\frac{x_k(n) - u_k(n)}{1 - c_{jk}(0)c_{kj}(0)} -$$

$$\frac{c_{kj}(0)\{x_j(n) - c_{jk}(0)[x_k(n) - u_k(n)] - u_j(n)\}}{[1 - c_{jk}(0)c_{kj}(0)]^2}] \quad (35)$$

$$\frac{\partial y_j(n)}{\partial c_{jk}(l)} = \frac{1}{1 - c_{jk}(0)c_{kj}(0)} \qquad (l \neq 0)$$

$$\times \left[\frac{\partial x_j(n)}{\partial c_{jk}(l)} - c_{jk}(0)\frac{\partial x_k(n)}{\partial c_{jk}(l)} + c_{jk}(0)\frac{\partial u_k(n)}{\partial c_{jk}(l)} - \frac{\partial u_j(n)}{\partial c_{jk}(l)}\right] \quad (36)$$

Let's define $\dot{y}_j(n, c_{jk}(l))$ as

$$\dot{y}_j\left(n, c_{jk}(l)\right) = \frac{\partial y_j(n)}{\partial c_{jk}(l)}, l = 0,1,\cdots,L_{jk} - 1 \quad (37)$$

The update equation of $c_{jk}(l)$ is expressed as follows:

$$c_{jk}(n+1, l) = c_{jk}(n, l) + \Delta c_{jk}(n, l) \quad (38)$$

$$\Delta c_{jk}(n, l) = -\mu\dot{q}\left(y_j(n)\right)\dot{y}_j(n, c_{jk}(l)) \quad (39)$$

5. SIMULATION

5.1. Simulation Conditions

Two-channel speech signals shown in Fig.5 are used for simulation. The two signals are sampled at 8 kHz.

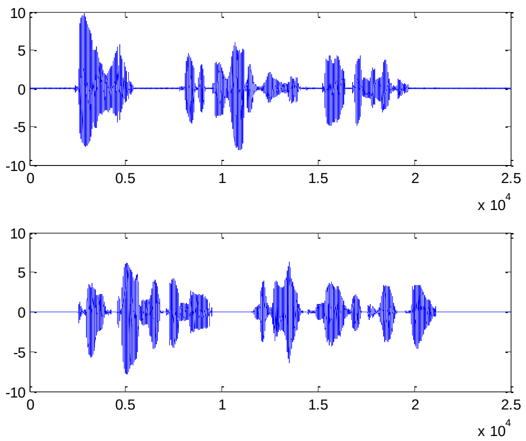

Fig.5: Waveform of signal sources.

The Convolutive mixing functions with 5 samples delay (Case-1) and no delay (Case-2) are used.

(Case-1)

$$H_{11} = 1 - 0.3z^{-T} + 0.14z^{-2T}$$
$$H_{12} = 0.5z^{-5T} + 0.2z^{-6T} + 0.015z^{-7T}$$
$$H_{21} = 0.5z^{-5T} + 0.125z^{-6T} + 0.005z^{-7T}$$
$$H_{22} = 1 - 0.33z^{-T} - 0.2z^{-2T}$$

(Case-2)

$$H_{11} = 1 - 0.3z^{-T} + 0.14z^{-2T}$$
$$H_{12} = 0.5 + 0.2z^{-T} + 0.015z^{-2T}$$
$$H_{21} = 0.5 + 0.125z^{-T} + 0.005z^{-2T}$$
$$H_{22} = 1 - 0.33z^{-T} - 0.2z^{-2T}$$

5.2. Evaluations

By using $\boldsymbol{P}(z)$ defined in Eq.(12), the separation performance is evaluated by the following SNR,

$$\sigma_s^2 = \sum_{i=1}^{2} \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|P_{ii}\left(e^{j\omega T}\right)\right|^2 d\omega T \quad (40)$$

$$\sigma_c^2 = \sum_{j \neq i} \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|P_{ji}\left(e^{j\omega T}\right)\right|^2 d\omega T \quad (41)$$

$$SNR = 10log\frac{\sigma_s^2}{\sigma_c^2}\ [dB] \quad (42)$$

σ_s^2 is the power of the selected signals, and σ_c^2 is that of the cross components.

5.3. Separation Performance

Figure 6 shows SNR using the FB BSS system and the FF-FB mixed BSS system under the conditions that delay time is 5 samples. The separation performances evaluated in SNR are approximately 16 dB for both methods. Although SNR of the FB BSS system is a little bit higher than that of the FF-FB mixed BSS system, the difference is quite small.

As is shown in Fig.7, when there is no delay in the mixing process, the separation performance of the FB BSS system deteriorates extremely and SNR is reduced to approximately 0 dB. This means that the signal sources cannot be separated from the mixture signals successfully. On the other hand, the FF-FB mixed BSS system has an excellent separation performance and SNR is improved to 22 dB, even higher than that under the delay conditions.

These results demonstrate that FF-FB mixed BSS system and the proposed learning algorithm are useful and robust for the transmission delay conditions in the mixing block.

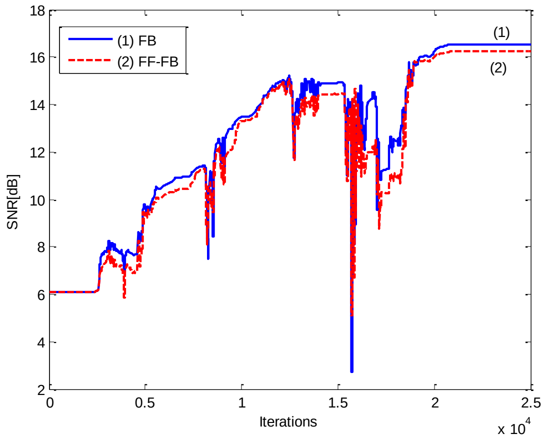

Fig.6: SNR with delay time = 5 samples

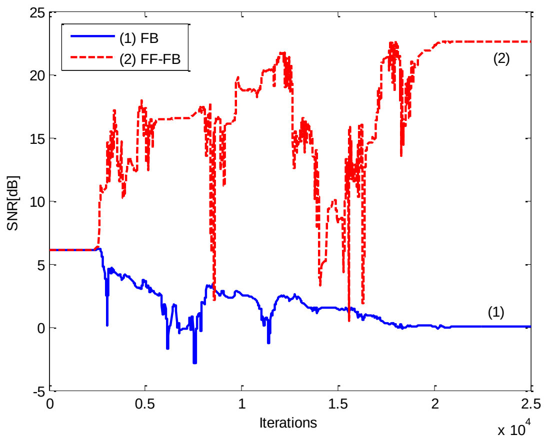

Fig.7: SNR with no delay

6. CONCLUSIONS

In this paper, we propose a learning algorithm for updating the coefficients of the FF-FB mixed BSS system [8]. The input signals, that is, the mixed signals, are applied to the FF-FB mixed BSS system, and its output signals are obtained. By using these output signals, the filter coefficients of the corresponding FB BSS system are updated. These coefficients are further transferred to the coefficients of the FF-FB mixed BSS system. This processing is carried out sample by sample. The simulation results for 2-channel case demonstrate the FF-FB mixed BSS system and the proposed learning algorithm are good for any transmission delay conditions in the mixing block.

REFERENCES

[1] C.Jutten and Jeanny Herault, “Blind separation of sourc-es, Part Ⅰ: An adaptive algorithm based on neromemitec architecture”, Signal Proc., 24, pp.1-10, 1991.

[2] P.Comon, C.Jutten and J.Herault, “Blind separation of s-Ources, Part Ⅱ: Problems statement”, Signal Processing, 24, pp.11-20, 1991.

[3] E.Sorouchyari, “Blind separation of sources, Part Ⅲ: Stability analysis”, Signal Proc., 24, pp.21-29, 1991.

[4] H.L.Nguyen Thi and C.Jutten, “Blind source separation for convolutive mixtures”, Signal Processing, vol.45, no.2, pp.209-229, March 1995.

[5] S.Haykin, Adaptive Filter Theory, 3rd ed., Prentice-Hall, Inc. 1996.

[6] K. Nakayama, A. Horita, A. Hirano, “A learning algorithm for convolutive blind source separation with transmission delay constraint,” Proc. IJCNN2002, pp.1287-1292, May 2002.

[7] K. Nakayama, A. Horita and A. Hirano, "Effects of propagation delays and sampling rate on feed-back BSS and comparative studies with feed-forward BSS," Proc. EUSIPCO2008, Lausanne, Switzerland, CD-ROM, 5 pages, Sept. 2008.

[8] A. Sugiyama, M. Matejko, “ A closed-form solution to the delay constraint in noise cancellation and feedback signal separation”. 24th Signal Processing Symposium, Kagoshima, pp.290-95, Nov. 2009.